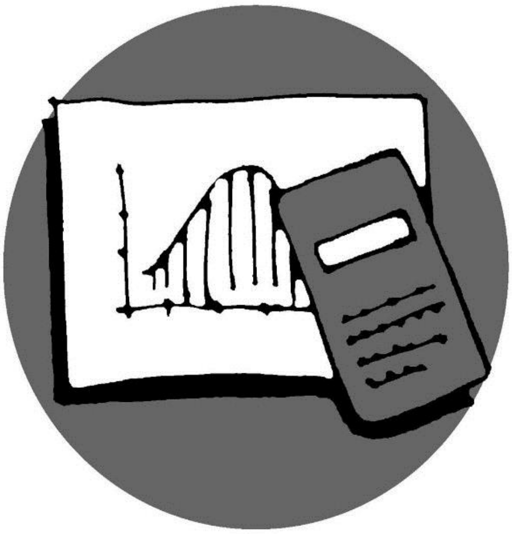
Library, Teaching and Learning

Analysis of Variance for Randomised Complete Block Design

(Two Way Analysis)



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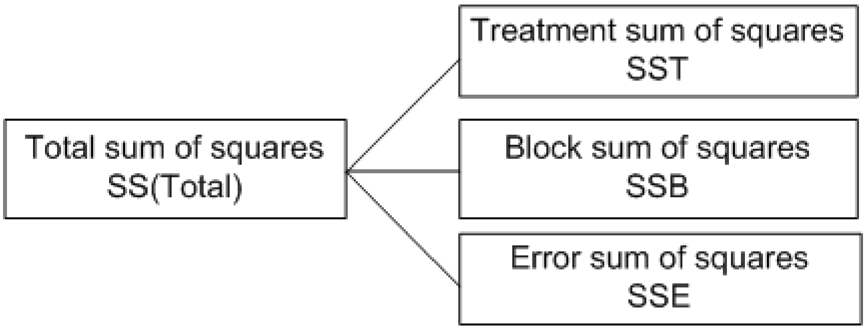
RANDOMISED COMPLETE BLOCK DESIGN

Two-way Analysis of Variance

Suppose in investigating the performance of the 3 types of cars, 5 drivers were each assigned one car of each type. In statistics, this is called a randomised complete block design, consisting of *b*=5 blocks, with *k*=3 treatments. Each block contains matched groups of *k* experimental units – one unit for each treatment - so in this example there are three experimental units in each block.

The analysis of variance for an RCBD partitions the total sum of squares into three parts:



The ANOVA table:

SOURCE	df	SS	MS	F	p
Blocks					
Treatment					
Error					
Total					

Note that the table is similar to the Completely Randomised ANOVA table, but there is now a row included for blocks. *b* is generally used to represent the number of blocks involved. Using this added information, the table can be completed:

ANOVA CALCULATIONS FOR RANDOMISED BLOCK DESIGN

The calculations are similar, but note the differences.

- Calculate the Degrees of Freedom for each source of variation:

$$df_{blocks} = (b-1); \; df_{treatment} = (k-1); \quad df_{total} = (n-1);$$
$$df_{error} = df_{total} - df_{treatment} - df_{blocks} = (n-k-b+1)$$

- Calculate the Correction Factor
- Calculate the Total Sum of Squares
- Calculate the Block totals
- Calculate the Treatment totals
- Calculate the Block Sum of Squares
- Calculate the Group (Treatment) Sum of Squares
- Calculate the Error Sum of Squares (by subtraction)
- Calculate the Mean Square for Block, Group and Error – divide $SS_{block}, \; SS_{treatmenet} \; and \; SS_{error}$ by $b-1, \; k-1 \; and (n-b-k+1)$ respectively.
- Calculate the F Value
- Determine the p-value

ANOVA Table for randomised complete block design

Correction Factor:	$CF = \frac{(\sum all\ entries)^2}{number\ of\ entries} = \frac{(\sum y)^2}{n}$	▪ Add all entries ▪ Square the result ▪ Divide by *n*
Total Sum of Squares:	$SS_{Total} = \Sigma(each\ \mathbf{entry})^2 - CF = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$	▪ Square each entry ▪ Add all results ▪ Subtract CF
Sum of Squares between Blocks:	$\mathrm{SS}_{Blocks} = \frac{\sum(Block\ total)^2}{number\ of\ treatments} - CF$ $= \frac{B_1^2 + B_2^2 + \cdots + B_k^2}{k} - \frac{(\sum y)^2}{n}$	• Square each block total • Add totals • Divide by no. of treatments • Add results • Subtract CF This *Sum of Squares* reflects the differences ***between*** the blocks
Sum of Squares between Treatments	$\mathrm{SS}_{Treatments} = \frac{\sum(Treatment\ total)^2}{number\ of\ blocks} - CF$ $= \frac{T_1^2 + T_2^2 + \cdots + T_k^2}{b} - \frac{(\sum y)^2}{n}$	▪ Square each group (Treatment) total ▪ Divide by no. of blocks ▪ Add results ▪ Subtract CF
Sum of Squares for Error:	$SS_{Error} = SS_{Total} - (SS_{Blocks} + SS_{Treatmentst})$	
Mean Square for Blocks:	$MS_{Blocks} = \frac{SS_{Blocks}}{b-1}$	
Mean Square for Treatments:	$MS_{Treatments} = \frac{SS_{Treatments}}{k-1}$	
Mean Square for Error:	$MS_{Error} = \frac{SS_{Error}}{(n-b-k+1)}$	
Test statistic *F*:	$F_{Treatments} = \frac{MS_{Treatments}}{MS_{Error}}$ $F_{Block} = \frac{MS_{Blocks}}{MS_{Error}}$	
Standard Error of Difference between means:	$SED(treatments) = \sqrt{\frac{2 \times MS_{Error}}{b}}$ Note General Formula: $SED = \sqrt{\frac{2 \times MS_{Error}}{r}}$	*b* is number of blocks *r* is number of values used to calculate the respective mean.

Returning to the Car Brand Problem:

	Driver					Total
	M	N	P	R	S	
Brand A	7.6	8.4	8.0	7.6	8.4	40.0
Brand B	7.8	8.0	9.1	8.5	9.6	43
Brand C	9.6	10.4	9.2	9.7	10.6	49.5
Totals	25	26.8	26.3	25.8	28.6	132.5

(i) $df_{block} = 4; df_{treatment} = 2; df_{total} = 14; df_{error} = df_{total} - df_{block} - df_{treatment} = 8$

(ii) Calculate the correction factor

$$CF = \frac{\left(\sum all\ entries\right)^2}{number\ of\ entries} = \frac{\left(\sum y\right)^2}{n} = 1170.42.$$

(iii) Calculate the Total Sum of Squares

$$SS_{Total} = \sum\left(each\ entry\right)^2 - CF = \sum y^2 - \frac{\left(\sum y\right)^2}{n} = 13.69$$

(iv) Calculate the Block Sum of Squares

$$SS_{block} = \frac{\left(25^2 + 26.8^2 + 26.3^2 + 25.8^2 + 28.6^2\right)}{3} - \frac{132.5^2}{15} = 2.427$$

(v) Calculate the Treatment Sum of Squares

$$= \frac{40^2}{5} + \frac{43^2}{5} + \frac{49.5^2}{5} - \frac{\left(\sum y\right)^2}{n} = 9.43$$

(vi) Calculate the Error Sum of Squares

$$SS_{error} = SS_{total} - SS_{block} - SS_{regression} = 13.69 - 2.427 - 9.43 = 1.833$$

(vii) Calculate

$$MS_{block} = \frac{2.427}{4} = 0.60675 \qquad MS_{treatment} = \frac{9.43}{2} = 4.715$$

$$MS_{error} = \frac{1.833}{8} = 0.229$$

(viii) Calculate F $F_{block} = \frac{0.60675}{0.229} = 2.64$ $F_{treatment} = \frac{4.715}{0.229} = 20.6$

SOURCE	df	SS	MS	F	p
Blocks	4	2.427	0.60675	2.65	*not significant*
Treatment	2	9.43	4.715	20.6	p<0.001
Error	8	1.833	0.229		
Total	14	13.69			

Practice questions

1. A Randomised Complete Block Design was used for an experiment on grapes to test the effect of bird repelling netting on the sugar content of the grapes at harvest. (The netting has some shading effect, and the beneficial effect of repelling birds has to be balanced against loss of production.) There were 8 blocks and 5 netting treatments (one of which is a control: that is no netting.) Fill out the following *part* of and an Analysis of Variance Table with the correct number of degrees of freedom for each of the appropriate sources of variance. (Note there may be more than two required here.)

Source of variance	Degrees of freedom

2. A trial examined the effectiveness of three types of insect traps. The three traps were set out five separate periods, which can be thought of as "blocks", and the average number of insects caught recorded.

   The Analysis of Variance table is:

Source of variance	Sums of Squares	DF	Mean Square	F ratio	P
Trap	427142	2			
Period	102715	4		2.64	
Error	77924	8	9740.5		
Total					

   What is the Mean Square for the Trap term?

3. In question 2, what is the p-value for the Period F-ratio?

4. In question 2, what is the Standard Error of the Difference (S.E.D.) between the Trap means?

5. Birth weight of babies born to mothers of various pregnancy weight groups (taken to be "blocks") and consumption of cigarettes (none, 1 pack/day, and more than 1 pack/day) were recorded.

   The data and the row and column totals are displayed as follows:
   `Rows: weight group Columns: cigarette group

1	2	3		All
1	3.2	2.8	1.7	7.7
2	3.2	2.8	2.5	8.5
3	3.2	3.1	2.5	8.8
4	3.4	3.1	2.6	9.1
5	3.5	3.3	2.8	9.6
6	3.5	3.4	2.9	9.8
All	20.0	18.5	15.0	53.5

(i) Complete the ANOVA table below

Analysis of Variance for birth weight

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*
Weight Group	0.983	5	0.1966	5.8	0.01
Cigarette Group	(a)	2	(d)	(e)	0.0001
Error	(b)	(c)	0.0339		
Total	3.516	17			

(ii) Calculate the standard error of the difference between the cigarette consumption means.

Answers Randomised Complete Block Design

1.

Source	df	working
Block	7	Block df = no. of blocks – 1 = 8 – 1 = 7
Netting	4	Netting df = no. of netting treatments–1 = 5 – 1 = 4
Error	28	Error df = Total df – Block df – Netting df = 39 – 7 – 4 = 28
Total	39	Total df = *n* – 1 = 40 – 1 = 39

2. $$MS_{trap} = \frac{SS_{trap}}{df_{trap}} = \frac{427142}{2} = 213571$$

3. P > 0.05.
Method: Look up critical values of F in tables for df1 = 4, and df2 = 8 for various values of α. $F_{.05,4,8}$ = 3.84 = 3.838 (depending on your table). Compare the tables F values with the calculated F (2.64 - from the ANOVA table in the question). Because $F_{calc} < F_{.05}$, conclude that P > 0.05.

4. $$sed(traps) = \sqrt{\frac{2 \times MS_{Error}}{b}} - \sqrt{\frac{2 \times 9740}{5}} = 62.4195$$ (No of periods =5)

5. (i) (a) $$SS_{cigarettes} = \frac{\Sigma(block^2)}{k} - CF = \frac{20^2 + 18.5^2 + 15^2}{6} - \frac{53.5^2}{18} = 2.194$$

(b) $$SS_{error} = SS_{total} - SS_{weight} - SS_{cigarettes} = 3.516 - 0.983 - 2.194 = 0.339$$

(c) df = 17 – 7 = 10

(d) $$MS_{cigarette} = \frac{2.194}{2} = 1.097$$

(e) $$F_{cigarette} = \frac{1.097}{0.0339} = 32.36$$

OR Start with (c) $df_{error} = 10 \quad \Rightarrow$

(b) $SS_{error} = 10 \times MS_{error} = 10 \times 0.0399 = 0.399$

$\Rightarrow$ (a) $SS_{cig} = SS_{total} - (SS_{weight} + SS_{error}) = 3.516 - (0.983 + 0.339) = 2.194$

$\Rightarrow$ (d) $MS_{cig} = \dfrac{SS_{cig}}{df_{cig}} = \dfrac{2.194}{2} = 1.097$

$\Rightarrow$ (e) as above.

(ii) $SED(cigarettes) = \sqrt{\frac{2 \times MS_{Error}}{b}} = \sqrt{\frac{2 \times 0.0339}{6}} = 0.1063$